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MATHEMATIC SIMULATION OF CUTTING UNLOADING FROM THE BUNKER

Serhii Yermakov
State Agrarian and Engineering University in Podilya, Ukraine
E-mail: ermkov@gmail.com

Taras Hutsol
State Agrarian and Engineering University in Podilya, Ukraine
E-mail: tarashutsol@gmail.com

Oleh Ovcharuk
State Agrarian and Engineering University in Podilya, Ukraine
E-mail: ovcharuk.oleh@gmail.com

Iryna Kolosiuk
State Agrarian and Engineering University in Podilya, Ukraine
E-mail: ikolosyk@ukr.net

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ABSTRACT

The peculiarities of cutting movement at unloading them from the hopper are described. The analysis of the scientific researches on bulk materials movement and bridging is given. To develop the mathematical model of cutting unloading the layer should be described as a pseudoliquid, that consists of discrete components (cuttings) and gaseous medium (air). The Navier-Stokes equation can be applied to the process of cutting unloading and velocity field. The equation of pseudoliquid motion is a nonlinear integral and differential equation. The initial and boundary conditions for speed of cutting movement are identified. As a result of research has been theoretically obtained a formula, that evaluates the rate of planting material unloading, the adequacy of which has already been partially tested in experimental experiments carried out by the authors on the way to creating an automatic planting machine.

Keywords: cutting; movement; mathematical model; pseudoliquid; modeling of motion; velocity field

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1. INTRODUCTION

Many machines that are used in production lines, deal with such workflows, as loading, unloading, handling, selection, transportation and etc. of different bulk materials. Efficiency and high-quality operation of such machines depends, primarily, on the speed of the unloading, the parameters and operation modes of handling devices (ADAMCHUK; BARANOV; BARANOVSKYI, 2001; VOITYUK; YACUN; DOVZHYK, 2008). The issue is complicated by the need to ensure a uniform and continuous unloading of material one size (length) of which is significantly higher than the other two sizes.

An example of such a material is cutting. This issue is an up-to-date one due to increased popularity of fuel on the basis of bioenergy crops and, consequently, the need in fast and efficient machines to create so called energy plantations. One of the most common crops is energy osier. The osier is planted by vegetative way with the help of cutting 20-25 cm long and 8-20 mm in diameter (graph 1).

Graph 1: Energy willow planting material

Planting is carried out by machines in which the planting material is fed manually (graph 2). This significantly limits the possibility of improving the efficiency of the units.

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Graph 2: Traditional energy willow planting with manual feed cuttings. Source: Willowpedia; Probstdorfer

To create a machine for planting such material the cutting should be transported fast and accurate. The study is an attempt to find strategies for justifying the cutting movement during their unloading from the accumulative capacity (YERMAKOV, 2017; YERMAKOV; BORYS, 2015; YERMAKOV; TULEJ; SHEVCHUK, 2018).

In general, the tasks associated with the loading and unloading of bulk and lump material operations, that are aimed at reduction of manual labor, at increase of performance and transport load factor, are of great importance. A lot of scientists and researchers focused on the issue of process stability in unloading material from the storage hopper.

To achieving positive results of their scientific research would not have been possible without the in-depth study on the patterns of granular material characteristics. The phenomenon of bridging takes place in the technological process of hoppers and vehicles with bodies of hopper construction. As a result, the time of complete cleaning of storage hopper and bulk cargo transportation is increased. This leads to violations of safety requirements in carrying out the activities and sufficiently large financial losses.

2. METHODS AND MATERIALS

It is obvious that this issue is connected not only with the raising of the level of technical and technological reliability of hopper devices, but also with the ensuring of the requirements of the occupational safety, health and the economic factor as well. The theoretical basis of the study is the researches of Ukrainian and foreign

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scientists on scientific methods of bulk material unloading from containers, issues of bridging problems and continuous flow of material.

To model the cutting movement in the middle of the hopper methods of hydrodynamics of multiphase systems is used. According to this approach, the combination of cutting is considered to be pseudoliquid that consists of two phases: a discrete phase formed by cuttings and continuous phase formed by gaseous medium (air). Each of these phases is regarded to be medium that allowed us to consider uploading as the movement of viscous incompressible pseudoliquid. The velocity speed of this pseudoliquid must satisfy Navier-Stokes equation.

3. RESULTS AND DISCUSSIONS

3.1. The process of granular materials floe in modern science and methodology

Numerous researches on bridging process made it possible to identify the dependencies that explain the essence of the process. The degree of influence of a huge number of various interrelated factors on bridging is difficult to assess and predict theoretically. It is about the geometry and hopper outlet and physical and mechanical properties of materials, and loading conditions, storage and release. It is due to the complexity of ensuring uniform continuous motion that excludes bridging, there is no universal feeding device, effectively working with any loose material.

At the same time the variety of material requires further contribution to the studies of motion of material. It is also difficult to overstate the scientific and practical value of the research on the mechanism of granular materials movement under its own weight because physical and mechanical properties of materials and their expiry date patterns have a decisive impact on the design of hoppers, as well as exhaust devices and mechanisms that enable expiration.

The researches of native scientists are devoted to physical and mechanical properties of bulk materials and patterns of their movement. Among the most significant the works of Bagnold (1954), Schulz (1967) Sokolovskiy (1954), Klein (1956), Sokolovskyi (1954), Zenkov (1964) should be considered. The works of Borshhev (2005), Dolgunin (2005), Gjachev (1992), Jenkins (1979), Protodjakonov (1981), Savage (1999), Yermakov (2018) and others deepened the previous

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researches. Their contribution to the theory of scientific basics of calculation theory involved the development of various unloading constructions.

Research on dynamics of bulk material flow from tanks, antibridging strategies and development of bridging equipment was done by the following scientists Bogomjagkih (1985, 2000), Geniev (1958), Gorjushinskyi (2003), Gyachev (1968, 1992), Jansen (1895), Keneman (1960), Kunakov (2000), Pepchuk (1985), Semenov (1980), Varlamov (2011), Zenkov (1964) et al. The basic characteristics, physical and mechanical properties of bulk materials that influence the bridging, research on the smooth functioning of hopper devices and development of equipment for bulk cargoes with a wide range of physical and mechanical properties are given in their work.

It should be noted that there is no unified theory in bulk material flow and processes of bridging in the hopper. For example, Zenkov (1964) note a significant influence of height of material layer on the velocity of bulk material flow.

Keneman (1960) determine the absence of such influence. In the process of storing chips in the hopper, they seal, which later after long storage leads to increasing adhesion between particles that in its turn decreases their mobility and promotes the growth of shear forces.

The experience of the hopper operation showed that Jansen formula underestimated the values of pressures on the bottom and walls of the hopper. This is due to the fact that the formula does not take into account the change in the density of the raw materials during storage in the hopper.

The theory by Jansen (1895) includes the case where the area of pressure is the whole bottom of the hopper, and lateral friction occurs between dissimilar bodies: loose body and the material of the hopper walls.

According to Jenike (1968) body is the union of homogeneous absolutely solid flat discs that are stacked in the correct rows.

The theory by Schulz (1967) considers cargo to be a bending beam. He pointed out that if homogeneous layers have little traction, each layer above goafing will bend itself under the influence of its own weight. Just like the loaded beam bents.

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Protodjakonov (1981) supposed that load on vault was considered vertical and distributed across its surface, and therefore a set of curve was parabolic. The issue of sets development in hoppers when the particle size of loose body is not too large in comparison with the size of the possible sets, is highlighted in the works by Vasilev (), but his research did not take into account the factors that influence the process of bridging.

The issue of adhesion between particles has been analyzed by Zenkov (1964). On the basis of stress state, he came to the conclusion that under certain conditions over the vent formed a set of matching paths from the greatest stress. The disadvantage of this theory is that it does not take into account the effect of overlying layers of loose material on the elementary volume selected over vent as well as the influence of the particle size.

Zenkov research on pressure distribution in bulk material was later developed in the monograph of Gjachev (1968). The scientist developed the differential equations of motion of elementary and ending amounts of granular material in hoppers of various shapes. The solution of these equations allowed to set the pressure distribution on the bottom and walls of the hoppers both in motion and calm state of loose materials. The theoretical dependency coincides with the experimental data for the dry crops, mineral fertilizers.

Adequacy by Gjachev model is explained, in particular, by the fact that parameters characterizing the bulk material: external and internal friction angles, angle stacking grains, grain size, etc. are included. This conclusion made it possible to study the effect of each option separately for granular material on pressure distribution laws. The proposed model allows scientists to explore some extreme cases.

According to Gjachev (1968) model, the infinitesimal amount of loose body grains turns into "liquid", which provides Coulomb friction between grains. In case of zero external and internal friction angles, this "fluid" turns into a so-called "perfect" liquid. It should be noted that the properties of such fluid differ from the usual properties of ideal fluid.

The study by Bogomjakih, Kunakov and Voronoj (2000) determines all phenomena that take place in the hopper in terms of static and dynamic state of

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granular materials on the basis of equivalent dynamic bridging. Analysis of the theoretical material shows that the bridging model of a loose body corresponds better to practical research.

Varlamov (2011) notes that the theory by Bogomjagkih (2000) describes the processes in bulk body when it is unloaded from the hoppers quite accurately, it reveals the theoretical basis for the relationship between the parameters of a loose body and accumulating parameters of the hopper, it allows providing the calculations of hopper systems. According to this theory dynamic and static bridging has a significant impact on the bulk movement of the body that is limited by the walls of the hopper. Dynamic bridging slows down the process of unloading and static stops it.

Analyzing the research papers by Alferov (1966), Zenkov (1966), Gjachev (1968), Bogomjagkih (1974), Semenov (1980) it may be noted that the main parameters that influence the flow of bulk materials from hoppers are: the size of hopper hole, physical and mechanical properties of bulk materials. The values of certain coefficients and constants are determined according to flowing material from the hopper and relations of constructive plan in terms of machines with big time performance are identified in a number of analytical dependencies. Therefore, the use of these dependencies in the hopper with the feeder in riddled installation with significantly reduced performance is not appropriate.

3.2. The preconditions for the implementation and the choice of technique process for unloading of cuttings from the hopper

There are disadvantages existing of granular materials. First of all, it is significant damage of the assembled product and the formation of the statically stable pickups. Second of all, increased speed of unload material with different physical and mechanical properties, etc. is the unsolved part of the research on machine productivity. One of the main obstacles in solving of this problem is the complexity of bridging.

Having analyzed the existing theories that reflect the essence of bridging, we may conclude that the majority of them describes the behavior of the material itself, but offers no solutions to the problems. In addition, since material properties vary greatly, it is obvious that there are no common approaches to solving the problems of bridging.

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In research papers of Gorjushinskyi (2003) the bridging is defined as the process of bridging development in containers in terms of loose cargo release.

The scientists distinguish two main areas to ensure smooth unloading of bulk cargo from tanks:

1. To prevent bridging that can be achieved by proper choice of parameters of capacity;
2. To destroy the formed arches with the help of different bridging devices.

Both directions are up-to-date ones. But the most progressive is the first one. It is better to prevent bridging, than to deal with it. The simulation of particle motion of loose material being unloaded, as well as the choice of means for the destruction of the bridging formed in the tank depend on the physical and mechanical properties of the material and the tank capacity.

This paper deals with the way the cuttings act in the process of unloading according to the influence of gravitational forces.

Let us assume that the layer of cuttings consists of circular cylinders with b length, $\overline{\rho}$ density and a radius. On the basis of research, we can conclude that during the pouring out of such bodies the issues connected with the position of cuttings in the longitudinal and transverse planes arise. However, the selection of parameters of an unloading device guaranties even and continuous motion of the material. To explore the process and build a mathematical model of movement of material it is important to determine the physical essence of a set of cuttings and determine the corresponding theory for describing her movements.

The process of unloading from the tank could be designed on the basis of methods for hydrodynamics of multiphase systems (SOUS, 1971; NYHMATULIN, 1978). According to this approach the total number cuttings that is influenced by the gravitational field and seismic fluctuations is modeled by a two-phase structure. This structure consists of discrete components (a set of cuttings) and continuous components (gaseous medium). These components in terms of mechanism of multiphase systems are treated as solid mediums.

These mediums are characterized by two effective coefficients of the viscosity caused by the interaction between cuttings and the interaction of cuttings with

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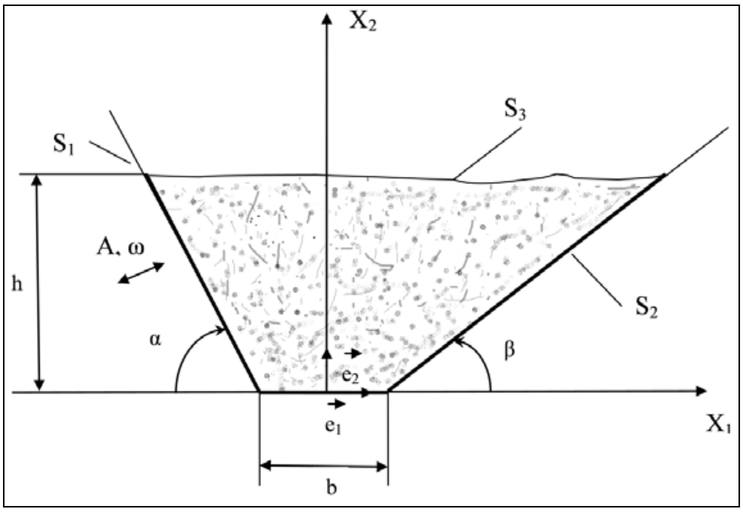
gaseous medium (air). We suppose that the bulk concentration of (discrete component) is bigger than similar values for continuous components. In this case the viscosity that is associated with the interaction with gas medium can be neglected. Therefore, the movement of discrete components can be designed as a movement of viscous incompressible pseudoliquid. The field rate of this pseudoliquid must satisfy Navier-Stokes equation.

The solution of this equation in the linear approximation is a mathematical process model of pouring out the cuttings from the hopper.

3.3. The model of the tank and two-phase pseudoliquid that models a set of cuttings

As a model for cutting unloading hopper we consider two half-planes that are located at α and β angles according to the horizontal plane. The width of the unloaded window can be identified as b. Let us introduce the Cartesian coordinate system x_1, x_2, x_3 with the axis x_3 that coincides with the line passing through the center of the unloaded window.



Graph 3: Design scheme of the hopper for cuttings

Graph 3 shows the cross section of the hopper at the platitude of $x_1 x_2$. Let us suppose that the movement of cuttings in the hopper does not depend on the coordinate x_3, i.e., we will focus on two-dimensional model of cutting unloading process. This limitation implies the existence of parallel plane walls that limit the movement of cutting along an axis. The combination of cutting in a hopper is considered to be influenced by the gravitational field and the vibrations. It is

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supposed that on one of the walls of the hopper is affected by harmonics with A amplitude and ω frequency. The direction of these vibrations is perpendicular to the wall of the hopper.

To simulate the movements of a set of cuttings the methods of hydrodynamic multiphase systems are used (SOUS, 1971; NYHMATULIN, 1978). According to this approach, we believe that the combination of cuttings is regarded as a pseudoliquid that consists of two phases: a discrete phase formed by cuttings and continuous phase - gaseous medium (air). Each of these phases is regarded as solid medium. Let us introduce the density of these solid mediums. $\overline{\rho}$ is considered to be the density averaged across all cuttings, $\overline{\rho}_1$ is the density of the gas (air) medium. Then the discrete phase density is determined by the formula (SOUS, 1971)

$$\rho = \delta\overline{\rho}, \tag{1}$$

and the density of continuous phase

$$\rho_1 = (1-\delta)\overline{\rho}_1. \tag{2}$$

Here δ - is the volumetric concentration of cuttings in the hopper. In addition to $\overline{\rho}$ and $\overline{\rho}_1$ parameters and pseudoliquid is characterized by efficient dynamic coefficients vibrio viscosity: μ is the coefficient of discrete phase viscosity that is characterized by the interaction between cuttings; μ_1 - is dynamic viscosity coefficient of gaseous medium. Let's assume that the volumetric concentration of cuttings is significantly more similar to values of continuous phase. In this case, the effective coefficient of vibrio viscosity that is determined by the interaction between cuttings and gaseous medium can be neglected. Thus, the movement of a set of cuttings will simulate a two-phase movement pseudoliquid.

3.4. Equation of motion of a two-phase pseudoliquid

Let us introduce the velocity of field: $\vec{V}$ is the field speed of discrete phase and $\vec{V}_1$ is the velocity field of the continuous phase. According to the works of Sous (1971), the equations of motion of a two-phase pseudoliquid can be represented in the following form

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$$\rho\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V}_1,\nabla)\vec{V}\right) = -\nabla P + \mu\Delta\vec{V} + \vec{F} - \rho g e_2\,, \tag{3}$$

$$\rho_1\left(\frac{\partial \vec{V}_1}{\partial t} + (\vec{V}_1,\nabla)\vec{V}_1\right) = -(1-\delta)\nabla P_1 + \mu_1\Delta\vec{V}_1 - \vec{F} - \rho_1 g e_2\,, \tag{4}$$

$$div\,\vec{V} = 0\,, \tag{5}$$

$$div\,\vec{V}_1 = 0\,, \tag{6}$$

e_1, e_2, e_3 are unit vectors of Cartesian coordinate system, P, P_1 – is discrete and continuous pressure phases, g is acceleration of free fall, $\vec{F}$ is the power that influences on mass unit of pseudoliquid and, according to the works of Sous (1971) has the form of:

$$\vec{F} = 0.5\frac{\rho\rho_1}{\bar{\rho}}\left[\frac{\partial}{\partial t}(\vec{V}_1-\vec{V}) + (\vec{V}_1-\vec{V},\nabla)(\vec{V}_1-\vec{V})\right] + 4.5\frac{\rho\rho_1\sqrt{\nu_1}}{\sqrt{\pi}a\bar{\rho}}\times$$
$$\int_0^t\left[\frac{\partial}{\partial t}(\vec{V}_1-\vec{V}) + (\vec{V}_1-\vec{V},\nabla)(\vec{V}_1-\vec{V})\right](t-\tau)^{-1/2}d\tau + \Phi(\vec{V}_1-\vec{V}), \tag{7}$$

where

$$\Phi = 37.5\frac{\vec{\rho}_1\nu_1\delta\rho}{\bar{\rho}a^2(1-\delta)^2}, \tag{8}$$

a is the radius of the circle that matches with an average area of cross-sections in cuttings, ν_1 is the coefficient of kinematic viscosity of continuous phase. Equations (3) to (6) are nonlinear integro-differential equations. The solution of these equations is possible only by numerical computer methods (KRYLOV; BOBKOV; MONASTYRSKYI, 1976). It follows from (3) in the process of hydrodynamic simulation of the cuttings movement, we should use such important characteristic as an effective coefficient of viscosity μ.

3.5. Initial and boundary conditions for speed movement of pseudoliquid

Mathematically correct statement about the flow of cuttings in equations (3) to (6) must be supplemented by the relevant initial and boundary conditions. Since the field velocity of a two-phase pseudoliquid $\vec{V}$, $\vec{V}_1$ and depend on a temporary

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variable, the initial conditions should be formulated. As a starting point of time, with no loss of generality, we choose time $t=0$. Let's assume that the initial time $\vec{V}$ and speed $\vec{V}_1$, pressure P and P_1 are equal to zero

$$\vec{V}\big|_{t=0} = \vec{V}_1\big|_{t=0} = 0, \tag{9}$$

$$\mathrm{P}\big|_{t=0} = P_1\big|_{t=0} = 0. \tag{10}$$

Since the movement of pseudoliquid occurs in a limited volume, we should put the appropriate boundary conditions on boundary surfaces of the hopper and free surface of cuttings that border with the atmosphere. In order to formulate these conditions, let us introduce stress tensors of discrete and continuous phases of pseudoliquid. According to the works of Sous (1971) and Nigmatulin (1978), we have

$$\sigma_{nm} = -P\delta_{nm} + \mu\left(\frac{\partial V_n}{\partial x_m} + \frac{\partial V_m}{\partial x_n}\right), \tag{11}$$

$$\sigma^1_{nm} = -P_1\delta_{mn} + \mu_1\left(\frac{\partial V_{1n}}{\partial x_m} + \frac{\partial V_{1m}}{\partial x_n}\right), \tag{12}$$

$m, n = 1,2.$

Here μ and μ_1 dynamic factors of viscosity in discrete and continuous phases of pseudoliquid.

$\vec{V} = V_1\vec{e}_1 + V_2\vec{e}_2, \vec{V}_1 = V_{11}\vec{e}_1 + V_{12}\vec{e}_2, \vec{e}_1, \vec{e}_2$ are unit vectors of Cartesian coordinate system (see graph 3). According to the established model of the hopper (see description above) equations describing its borders, have a look

$$S_1 : x_2 = -tg\alpha(x_1 + b/2), \quad x_1 \le -b/2, \tag{13}$$

$$S_2 : x_2 = tg\beta(x_1 - b/2), \quad x_1 \ge b/2. \tag{14}$$

At these borders, taking into account the slippage of cuttings that form the discrete phase, equal tangents of discrete phase must be performed according to resistance force σ_τ of discrete phase per unit area of hopper boarders.

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According to relatively free surface let's assume the process of cuttings motion remain flat. Then the equation can be represented as

$$S_3 : x_2 = h(t), \tag{15}$$

where $h(t)$ function depends only on the temporary variable and its value coincide with the distance from the free surface to plane unloaded window.

At S_3 free surface, you must meet the following boundary conditions. We will neglect the influence of the atmosphere on the dynamics of pseudoliquid. Then on the free surface of the discrete voltage phases must apply to zero

$$\vec{n}\sigma\vec{n} = 0, \tag{16}$$

$$\vec{n}\sigma\vec{t} = 0. \tag{17}$$

Here $\vec{n}$ and $\vec{t}$ are normal unit vector and tangent unit vector to the free surface.

In addition to the conditions (16), (17) so called kinematic boundary conditions must be carried out. On S_3 free surface the velocity field of discrete phases must satisfy the condition

$$\frac{\partial F}{\partial t} + \left(\vec{V}, \nabla\right)F = 0, \tag{18}$$

where $F(t, x_1, x_2) = x_2 - h(t)$.

Let's transform conditions (16), (17) and (18). Vectors $\vec{n}$ and $\vec{t}$ have the appearance of

$$\vec{n} = \vec{e}_2, \vec{t} = \vec{e}_1. \tag{19}$$

Substituting (19), (16) and (17), we have

$$\sigma_{22} = 0, \tag{20}$$

$$\sigma_{21} = 0. \tag{21}$$

From (20), (21) with the help of (11), (12) we get

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$$-P+2\mu\frac{\partial V_2}{\partial x_2}=0, \quad (22)$$

$$\frac{\partial V_1}{\partial x_2}+\frac{\partial V_2}{\partial x_1}=0. \quad (23)$$

These ratios must be executed when $x_2-h(t)=0$. Next (18) have.

$$\dot{h}=V_2, \quad (24)$$

where the dot indicates operation of differentiation

Thus, on S_3 free surface of velocity field the discrete phases of pseudoliquid must satisfy conditions (22) to (24).

According to $\vec{V}_1$ velocity of continuous phase let's assume that the same conditions are met. Let us now consider boundary conditions on S_1 and S_2 walls of a hopper. Let's believe that S_1 boundary is exposed to harmonic oscillation with A amplitude and ω frequency. The direction of these fluctuations coincides with the direction of the unit vector to S_1 (24).

Then the normal components of $\vec{V}$ velocity of the discrete phase on S_1 border must satisfy condition.

$$\left(\vec{n}_1,\vec{V}\right)=A\omega\sin\omega t, \quad (25)$$

where unit normal vector is $\vec{n}_1=\sin\alpha\vec{e}_1+\cos\alpha\vec{e}_2$.

condition (25) can be represented as

$$\sin\alpha V_1+\cos\alpha V_2=A\omega\sin\omega t. \quad (26)$$

This condition must be met when $x_2=-tg\alpha(x_1+b/2),-b/2-h_0ctg\alpha\le x_1\le -b/2$.

Parallel with the condition (26) $\vec{V}$ velocity must meet the condition of equality of tangential stresses to force resistance of $S_1(S_2)$ surface movement of discrete phase, transported to middle square

$$\vec{n}_1\sigma\vec{t}_1=F_{S_1}, \quad (27)$$

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$$\vec{n}_2 \sigma \vec{t}_2 = F_{S_2}. \tag{28}$$

Here

$$\vec{n}_1 = \sin\alpha \vec{e}_1 + \cos\alpha \vec{e}_2, \vec{t}_1 = \cos\alpha \vec{e}_1 - \sin\alpha \vec{e}_2, \tag{29}$$

$$\vec{n}_2 = -\sin\beta \vec{e}_1 + \cos\beta \vec{e}_2, \vec{t}_2 = \cos\beta \vec{e}_1 + \sin\beta \vec{e}_2, \tag{30}$$

where $\vec{n}_1, \vec{t}_2$ are vectors that are perpendicular and tangent to S_2 surface.

Let's substitute the expression (11) for stress tensor in (27), (28) and will perform the necessary calculations.

Will have:

$$\mu \cos 2\alpha \left(\frac{\partial V_1}{\partial x_2} + \frac{\partial V_2}{\partial x_1} \right) + 2\mu \sin 2\alpha \frac{\partial V_1}{\partial x_1} = F_{S_1}, \tag{31}$$

$$\mu \cos 2\beta \left(\frac{\partial V_1}{\partial x_2} + \frac{\partial V_2}{\partial x_1} \right) - 2\mu \sin 2\beta \frac{\partial V_1}{\partial x_1} = F_{S_2}. \tag{32}$$

F_{S_1} and F_{S_2} resistance forces according to the works of Tyshchenko (2011), and if we assume that S_1 and S_2 borders and are smooth, coincide with the pendant dry sliding

$$F_{S_1} = fN_1, F_{S_2} = fN_2, \tag{33}$$

where f is the coefficient of friction, $N_{1,2}$ is the normal pressure.

Normal pressure of discrete phases on S_1 and S_2 surfaces can be calculated by using formulas

$$N_1 = \frac{\rho g \cos^2 \alpha h(t)}{2}, N_2 = \frac{\rho g \cos^2 \beta h(t)}{2}. \tag{34}$$

Let's plug (34) in (31) and (32) and finally get the boundary conditions at S_1 and S_2 borders in the hopper

$$\cos 2\alpha \left(\frac{\partial V_1}{\partial x_2} + \frac{\partial V_2}{\partial x_1} \right) + 2\sin 2\alpha \frac{\partial V_1}{\partial x_1} = f \frac{g \cos^2 \alpha h(t)}{2\nu}, \tag{35}$$

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$$\cos 2\beta\left(\frac{\partial V_1}{\partial x_2}+\frac{\partial V_2}{\partial x_1}\right)-2\sin 2\beta\frac{\partial V_1}{\partial x_1}=f\,\frac{g\cos^2\beta h(t)}{2\nu}. \tag{36}$$

Here $\nu = \mu/\rho$ is the kinetic coefficient vibro viscosity. Conditions (35) and (36) must be carried out when

$$x_2 = -tg\alpha\left(x_1 + b/2\right), - h_0 ctg\alpha - b/2 < x_1 - b/2$$ и

$$x_2 = tg\beta\left(x_1 - b/2\right), b/2 < x_1 < b/2 + h_0 ctg\beta,$$

where $h_0 = h(0)$ is the maximum thickness of the cutting layer at $t = 0$. zero time.

Consequently, the velocity of two-phase of pseudoliquid that stimulates a set of cuttings must satisfy the initial conditions (9), (10) and (2) to (4), (26), (35), (36) boundary conditions.

4. CONCLUSION AND RECOMMENDATIONS

The review of information sources showed that the research on the regularities of the mechanics of granular body flow deals with a number of theoretical and experimental research. The analyzed scientific issues are quite complex and are covered with the help of classical mechanics, theory of plasticity, soil mechanics and rheology on the basis of mathematical modeling. Moreover, the majority of papers explore of small-sized materials, and process of unloading of such materials as cuttings hasn't been studied yet.

To simulate the movements of a set of cuttings the hopper model was presented as two half-planes located at α and β angles to the horizontal plane. The width of the unloaded window will be marked as b. For mathematical processing methods of hydrodynamics of multiphase systems in which the totality of cuttings is regarded as a pseudoliquid and consists of two phases: discrete phase formed by cuttings and continuous phase-gaseous medium (air) were used.

The patterns of movements that are nonlinear integra differentiate equations, which can be processed on the basis of software, are presented for this pseudoliquid. Two-phase velocity field of pseudoliquid model the combination of cuttings, must meet the primary and regional conditions, which are presented in the

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form of equations. As the result, the preconditions for the establishment of mathematical model for unloading the layer of cuttings from the hopper are created in the study.

Mathematical simulation of energy willow cutting unloading from the hopper allows theoretically substantiating the possibility of increasing the planting process efficiency, up to its full automation. As a result of research has been theoretically obtained a formula, that evaluates the rate of planting material outflow, the adequacy of which has already been partially tested in experimental experiments carried out by the authors on the way to creating an automatic planting machine.

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